Lecturer Math Past Paper PPSC 2017

Q.1 The differential equation $(1-x^2)y''-2xy'+n(n+1)y=0$ is:

(A) Bessel Equation

(B) Legendre equation

(C) Poisson equation

(D) None of these

Solution:

Bessel equation: $x^2y''+xy'+(x^2-v^2)y=0$

Legendre equation: $(1-x^2)y''-2xy'+n(n+1)y=0$

Poisson equation: $u_{xx}+u_{yy}=f(x,y)$

Q.2 If a and b are parallel or anti-parallel, then:

(A) $\underline{a}\times\underline{b}=0$ (B) $\underline{a}\times\underline{b}=-\underline{b}\times\underline{a}=b$

(C) $\underline{a}\times\underline{b}=0$ (D) None of these

Solution:

When two vectors are parallel the angle $0°$ between them when anti-parallel then $-180°$

In both case $\underline{a}\times\underline{b}=ab\sin\theta=ab(0)=0$

Q.3 The vector equation of line through a point with position vector a and a parallel vector b (t is a scalar):

(A) $t^2=\overrightarrow{ra}+\vec{b}$

(B) $\vec{r}=\vec{a}+t\vec{b}$

(C) $\vec{r}=(1-t)\vec{a}+t\vec{b}$

(D) None of these

Solution:

If r_0 is a position vector of the point P_0 then the line and v is a parallel vector must have the form $\vec{r}=\overrightarrow{r_0}+t\vec{v}$

Q.4 The angle between the vector is $\frac{\pi}{4}$ $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}\frac{\vec{a}\times\vec{b}}{\vec{a}\cdot\vec{b}}$ is equal to:

(A) 1 (B) $\frac{1}{\sqrt{2}}$

(C) $\sqrt{\frac{5}{2}}$ (D) $\frac{1}{2}$

Solution:

$$\underline{a}\times\underline{b}=ab\sin\frac{\pi}{4}=ab\left(\frac{1}{\sqrt{2}}\right)$$

$$\underline{a}\cdot\underline{b}=ab\cos\frac{\pi}{4}=ab\left(\frac{1}{\sqrt{2}}\right)$$

$$\frac{\vec{a}\times\vec{b}}{\vec{a}\cdot\vec{b}}=1$$

Q.5 If a and b are unit vectors and θ is the angle between them, then the value of $\left|\cos\frac{\theta}{2}\right|$ is:

(A) $\frac{1}{2}|a+b|$ (B) $\frac{1}{2}|a-b|$

(C) $\left|\frac{a-b}{a+b}\right|$ (D) $\left|\frac{a+b}{a-b}\right|$

Solution:

$$|\vec{a}|=1 \quad \& \quad |\vec{b}|=1$$

$$|\vec{a}+\vec{b}|^2=\vec{a}^2+\vec{b}^2+2\vec{a}\vec{b}$$

$$=1+1+2ab\cos\theta$$

$$=2+2\cos\theta$$

As $\cos 2\theta^2=\frac{1+\cos 2\theta}{2}$

$$2\cos^2\theta=1+\cos 2\theta$$

$\Rightarrow 2\cos^2\frac{\theta}{2} = 1 + \cos\theta$

So

$$|\vec{a}+\vec{b}|^2 = 2(1+\cos\theta)$$

$$|\vec{a}+\vec{b}|^2 = 2(2\cos^2\frac{\theta}{2})$$

$$|\vec{a}+\vec{b}|^2 = 4\cos^2\frac{\theta}{2}$$

Taking square root

$$|\vec{a}+\vec{b}| = 2\cos\frac{\theta}{2}$$

$$\frac{1}{2}|\vec{a}+\vec{b}| = \cos\frac{\theta}{2}$$

Q.6 The work done by a force $\vec{F} = 2\hat{i} - \hat{j} - \hat{k}$ through a displacement $\vec{r} = 3\hat{i} + 2\hat{j} - 5\hat{k}$ is:

(A) 3 (B) 6

(C) 9 (D) 12

Solution:

$$W = \vec{F}\cdot\vec{r}$$

$$= (2\hat{i} - \hat{j} - \hat{k})\cdot(3\hat{i} + 2\hat{j} - 5\hat{k})$$

$$= 6 - 2 + 5 = 9$$

Q.7 Any three vectors $\vec{A}, \vec{B}$ and $\vec{C}$ are coplanar if, and only if:

(A) $\vec{A}\cdot(\vec{B}\times\vec{C}) = 0$ (B) $\vec{A}\times(\vec{B}\cdot\vec{C}) = 0$

(C) $\vec{A}\cdot(\vec{B}\cdot\vec{C}) = 0$ (D) $\vec{A}\times(\vec{B}\times\vec{C}) = 0$

Solution:

Condition of vector coplanarity

- **For three vector:** The three vectors are coplanar if there scalar triple product $(\vec{A}\cdot(\vec{B}\times\vec{C}) = 0)$ is zero
- **For Three or n vectors:** The three or n vectors are coplanar if among them no more than two are linearly dependent

Q.8 The vectors $2\hat{i} - \hat{j} + \hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ are:

(A) parallel (B) Collinear

(C) Coplanar (D) None of these

Solution:

$\vec{a}\times\vec{b} = 0$ Then vector are parallel or anti-parallel.

Definition: Two Vector parallel to one line or lying on one line are called **collinear vectors.**

Condition of collinearity:

$$\vec{a} = n\cdot\vec{b}$$

Two vectors are collinear if relation of there coordinates is equal.

Two vectors are collinear if there cross product is equal to zero.

Condition of coplanarity of two vectors:

If $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and $\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$ are two vectors said to be coplanar iff $\begin{bmatrix} a_1 & a_2 & 1 \\ b_1 & b_2 & 1 \\ c_1 & c_2 & 1 \end{bmatrix} = 0$

$$\begin{bmatrix} 2 & 3 & 1 \\ -1 & -4 & 1 \\ 1 & -4 & 1 \end{bmatrix} = 0$$

Q.9 If a and b are two vectors such that $\underline{a}\cdot\underline{b} = 0$ and $\underline{a}\times\underline{b} = 0$, then:

(A) a and b are parallels

(B) a and b are mutually perpendicular

(C) a and b are both zero

(D) None of these

Solution:

$$\underline{a}\cdot\underline{b}=0$$

$$\underline{a}\cdot\underline{b}=ab\cos=0\Rightarrow\theta=90$$

&

$$\underline{a}\times\underline{b}=0$$

$$\underline{a}\times\underline{b}=ab\sin\theta=0\Rightarrow\theta=0$$

⇒ vector is not parallel not perpendicular

⇒ a and b are both zero

Q.10 The volume of the parallelepiped with side $\overline{A}=6\hat{i}-2\hat{j}$, $\overline{B}=\hat{j}+2\hat{k}$ $\overline{C}=\hat{i}+\hat{j}+\hat{k}$ is:

(A) 3cubic (B) 10 cubic

(C) 15 cubic (D) 20 cubic

Solution:

Volume of parallelepiped $=\overline{A}\cdot(\overline{B}\times\overline{C})$

$$V=\begin{vmatrix}6 & -2 & 0\\ 0 & 1 & 2\\ 1 & 1 & 1\end{vmatrix}=10\text{cubic}$$

Q.11 A particle of mass m moves in a circle of radius r with constant speed v, the force F acting on particle is:

(A) $F=\frac{mv}{r^2}$ (B) $F=\frac{mv^2}{r^2}$

(C) $F=\frac{mv^2}{r}$ (D) None of these

Solution:

Centripetal force $=F=\frac{mv^2}{r}$

Q.12 The centre of mass of a thin rod of a length "l"

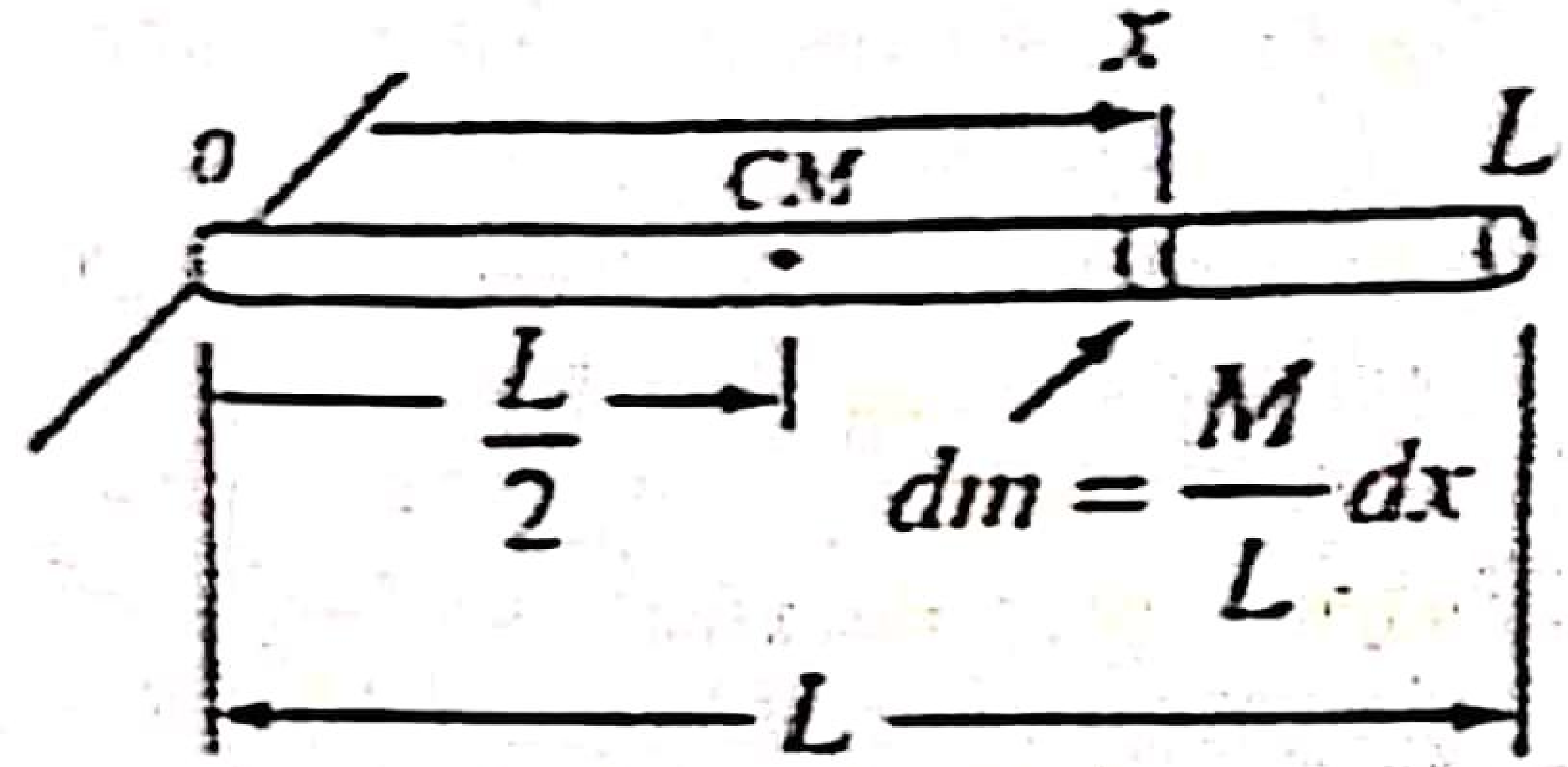
(A) $\frac{1}{4}$ (B) $\frac{1}{3}$

(C) $\frac{1}{2}$ (D) None of these

Solution:

$$x_{cm}=\frac{\int_0^M x\,dm}{M}=\frac{\int_0 x\frac{M}{L}dx}{M}$$

$$=\frac{1}{L}\frac{x^2}{2}\Big|_{x=0}^{x=L}=\frac{L}{2}$$



Q.13 The centre of mass of a semi-circular lamina $x^2+y^2=a^2$ in the upper half lie on :

(A) The origin (B) x-axis

(C) y-axis (D) None of these

Solution:

A lamina is a 2-dimension object. In other word a flat object whose thickness can be ignore.

If a body has a line of symmetry, the center of mass will be lie on the line. This fact is very useful.

For example, the center of mass of a circular lamina is at the center of circle, since the center of mass is on each each axis of symmetry and they all meet at the center.

Results

- The centre of mass of a uniform semi-circular lamina of radius r lie on the axis of symmetry at the distance of $\frac{4r}{3p}$ from the straight edge.
- The centre of mass of a uniform solid right circular cone of height h lie on the axis of symmetry at the distance of $\frac{h}{4}$ from the base.
- The centre of mass of a uniform solid hemisphere of radius r lie on the axis of symmetry at the distance of $\frac{3r}{8}$ from the base

So lie on upper half and symmetric axis is y-axis

Q.14 The moment of inertia of uniform solid sphere of radius a and mass M about a diameter is:

(A) $\frac{1}{2}Ma^2$ (B) $\frac{1}{3}Ma^2$

(C) Ma^2 (D) $\frac{2}{5}Ma^2$

Solution:

- The inertia thin spherical shell about symmetry axis is $I = \frac{2}{3}MR^2$
- The inertia of sold cylinder or disc about symmetry axis is $I = \frac{1}{2}MR^2$
- The inertia of sold cylinder about central diameter is $I = \frac{1}{4}MR^2 + \frac{1}{12}MR^2$
- The inertia of hoop about symmetry axis is $I = MR^2$
- The inertia of hoop about diameter is $I = 1/2MR^2$
- The inertia of solid sphere about symmetry axis is $I = \frac{2}{5}MR^2$
- The inertia of solid sphere about symmetry axis is $I = \frac{2}{5}MR^2$
- The inertia of rod of length about centres is $I = 1/12ML^2$
- The inertia of rod of length about end is $I = \frac{1}{3}ML^2$

Q.15 The radial and transverse velocity of a particle be non-zero constants, then the path described by the particle is:

(A) Ellipse (B) A spiral

(C) A circle (D) None of these

Solution:

The radial and transverse. Velocity related to the motion of stars the move a circular path the radial velocity refer to the motion along y-axis and transverse refer to motion along x-axis

Q.16 The path described by a particle moving with zero velocity and acceleration is a:

(A) Circle (B) Straight line

(C) point (D) None of these

Solution:

When a body is moving in a circular path its direction changing with motion so its velocity consider zero.

Q.17 A stone of 4kg falls freely from rest at the height of 4.9m. The momentum of the body when strike the ground is:

(A) 39.59 kg m/s (B) 39.8kg m/s

(C) 39.2kg m/s (D) None of these

Solution:

$$2gh = v_f^2 - v_i^2$$
$$2(10)4.9 = v_f2 - 0$$
$$98 = v_f2$$
$$7\sqrt{2} = v_f$$
$$P = mv$$
$$P = 4 \times 7\sqrt{2} = 39.59$$

Q.18

Q.19

Q.20

Q.21 The minimum velocity of a body to escape from gravitational field is equal to

(A) $\sqrt{2g}$ (B) $\sqrt{2gR}$

(C) $\sqrt{\frac{g}{R}}$ (D) None of these

Solution:

$$V_{esc} = \sqrt{2gR}$$

Q.22 If F is the magnitude of friction, R that of normal reaction and μ the coefficient of friction, then:

(A) $FR = \mu$ (B) $F = \mu R$

(C) $F = \mu\sqrt{R}$ (D) None of these

Solution:

$$\mu = \frac{F}{R}$$
$$F = \mu R$$

Q.23 The power set P(X) of a non empty set X is

(A) The smallest topology on X

(B) The largest topology on X

(C) Not a topology

(D) None of these

Solution:

Discrete Topology: The power set P(X) of a non empty set X is called the discrete topology it is the largest topology on setX.

Q.24 If τ_1 and τ_2 are two topologies on X such that $\tau_1 \subset \tau_2$ then:

(A) τ_1 is said to be finer than τ_2

(B) $\tau_1 \subset \tau_2$

(C) $\tau_1$is said to be coarser than τ_2

(D) None of these

Solution:

If τ_1 and τ_2 are two topologies defined on the non empty set X such that $\tau_1 \subset \tau_2$ then τ_1 is said to be coarser or weaker than τ_2 and τ_2 is said to be finer or stranger than τ_1

Q.25 A point $x \in A$ is said to be an interior point of A if there exists an open set U that

(A) $(A \cap U)\setminus\{x\}=\emptyset$ (B) $A \cap U = \emptyset$

(C) $U \subset A$ (D) None of these

Solution:

Interior Point of a Set: Let (X, τ) be the topological space and $A \subset X$ then a point $x \in A$ is said to be an interior point of set A, if there exists an open set U such that $x \in U \subset A$

Q.26 A subset of a topological space X is said to be dense in X if:

(A) $\overline{A} = A$ (B) $\overline{A} = \emptyset$

(C) $\overline{A} = X$ (D) None of these

Solution:

Dense: A subset A of a topological space X is dense whose closure is entire space X

$$\overline{A} = X$$

Q.27 A point x of topological space X is said to be the limit point of a subset A of X, if for every open set U containing x we have

(A) $(A \cap U)\setminus\{x\} = \emptyset$

(B) $A \cap U = \emptyset$

(C) $(A \cap U)\setminus\{x\} \neq \emptyset$

(D) None of these

Solution:

Limit Point of a Set: Let (X, τ) be the topological space and $A \subset X$ then a point $x \in X$ is said to be a limit point or accumulation point or cluster point of set A, if there exists an open set U and $x \in U$ contain at least one point of A different from x

In other word $\{A \cap U\}\setminus\{x\} \neq \emptyset$

Q.28 Which of the following statement is true

(A) A limit point of A is always a point of A

(B) A limit point of A is always limit point of A

(C) A limit point of A is always limit point of A

(D) A limit point of A is always limit point of A

Solution:

Limit Point of a Set: Let (X, τ) be the topological space and $A \subset X$ then a point $x \in X$ is said to be a limit point or accumulation point or cluster point of set A, if there exists an open set U and $x \in U$ contain at least one point of A different from x

In other word $\{A \cap U\}\setminus\{x\} \neq \emptyset$

It is clear from the above that limit point of set A may or may not be the point of A

Q.29 A point $x \in A$ is said to be an isolated point of A if there exists an open set U that

(A) $A \cap U = \{x\}$ (B) $A \cap U = \{x\}$

(C) $A \cap U = \{x\}$ (D) $A \cap U = \{x\}$

Solution:

Isolated Point of a Set: Let (X, τ) be the topological space and $A \subset X$ then a point $x \in A$ is said to be an isolated point of set A, if there exists an open set U and $x \in U$ such that $A \cap U = \{x\}$

Mean does not contain any other point of A different than x

Q.30 Which of the following statement is true

(A) An isolated point of A is always limit point of A

(B) An isolated point of A can never be limit point of A

(C) A limit point of A is always limit point of A

(D) A limit point of A is always limit point of A

Solution:

Isolated point definition $A \cap U = \{x\}$

Limit point define $\{A \cap U\}\setminus\{x\} \neq \emptyset$

From the both definition it is clear that an isolated point of A can never be limit point of A

Q.31 Let $X = \{a, b, c, d, e\}$ with topology $\tau = \{\emptyset, \{b\}, \{a, d\}, \{a, b, d\}, \{a, c, d, e\} X\}$, if $A = \{a, b, c\}$

(A) $A^{\circ} = \{a\}$ (B) $A^{\circ} = \{b\}$

(C) $A^{\circ} = \{b\}$ (D) None of these

Solution:

The element of τ are opens interior of A is the biggest open set from τ which contain in A

So $A^{\circ} = \{b\}$

Q.32 The sequence $\{x_n\}$ of points of a metric space X is said to be convergent to $x \in X$ for each $\varepsilon > 0$, there exists a positive integer n_0 such that $n \geq n_0 \rightarrow$

(A) $d(x_n, x) < \varepsilon$ (B) $d(x_n, x) = \varepsilon$

(C) $d(x_n, x) > \varepsilon$ (D) None of these

Solution:

Definition of convergence sequence in metric space: The sequence $\{x_n\}$ of points of a metric space X is said to be convergent to $x \in X$ for each $\varepsilon > 0$, there exists a positive integer n_0 such that $n \geq n_0 \rightarrow d(x_n, x) < \varepsilon$.

Q.33 Area of the parallelogram with base x and attitude y is

(A) xy (B) $x + y$

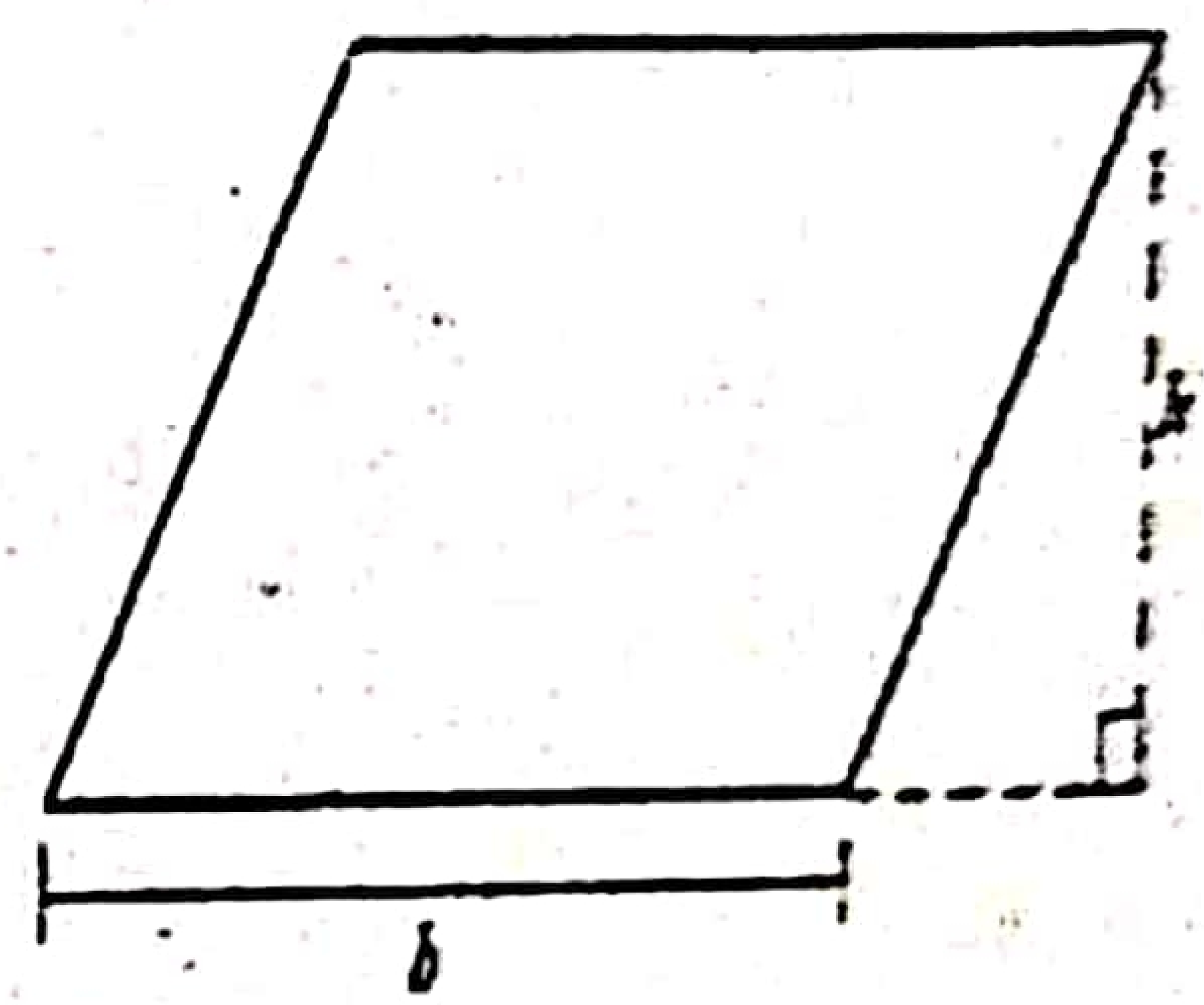
(C) $2x + 2y$ (D) None of these

Solution:

The area A of a parallelogram is given by the formula

$A = bh$,

where b is the length of one base and h is the height. (The figure below shows how to find the height of a parallelogram.)



Q.34 are the length of sides of:

(A) Right triangle

(B) Equilateral triangle

(C) Obtuse triangle

(D) None of these

Solution:

Right triangle: A triangle with one angle is 90°.

Equilateral triangle: A triangle with all three sides of equal length and all angles are 60°.

Oblique Triangle: A triangle with no right angle with three acute angles.

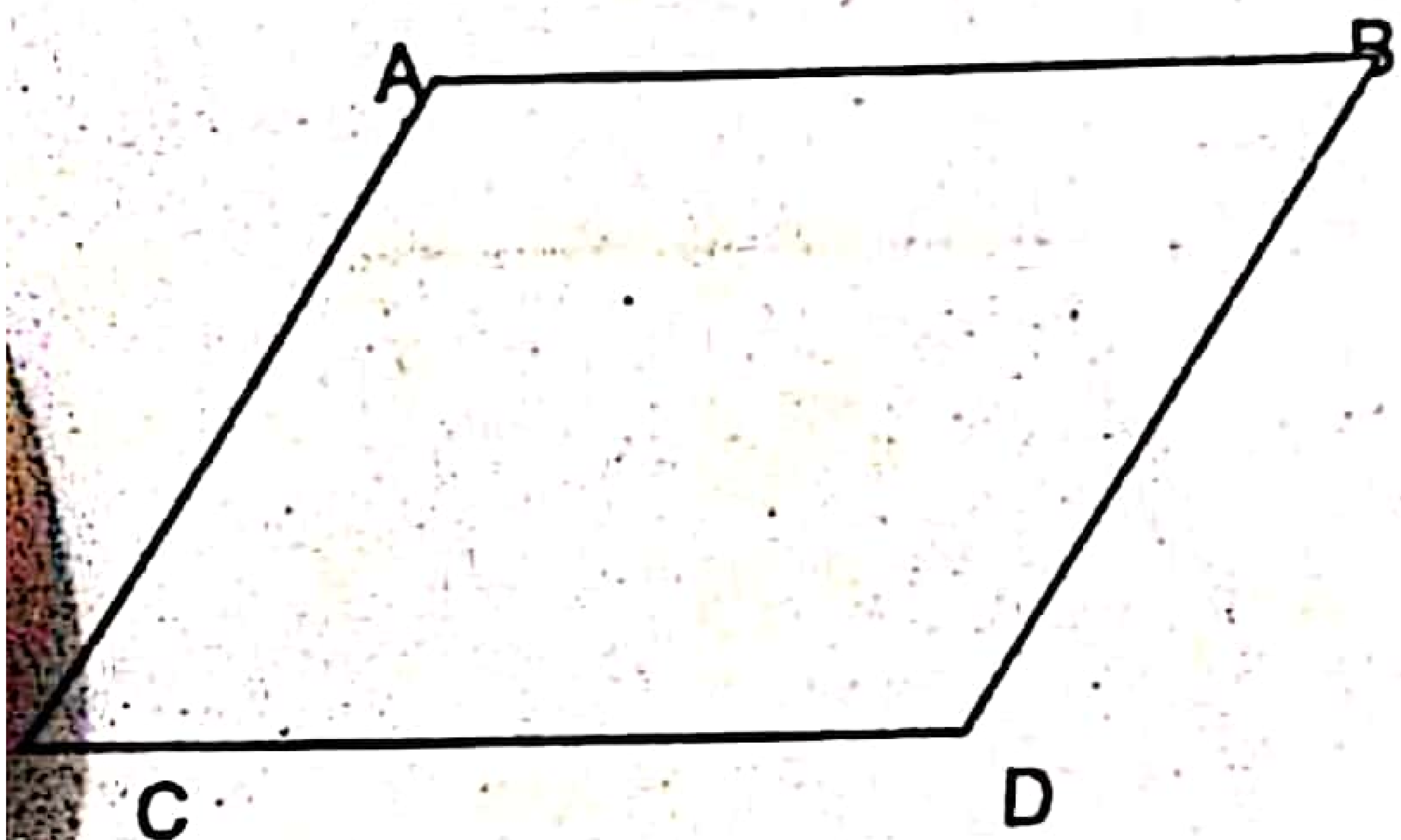
Obtuse Triangle: A triangle that has an angle greater than 90°

Q.35 The external angle of the $\parallel$ gram 120°, then the adjacent internal angle is:

(A) 120° (B) 50°

(C) 60° (D) None of these

Solution:



The angle at D is opposite angle of angle at B.

The angle at C opposite angle of angle at A.

$\angle A = \angle C$ & $\angle B = \angle D$

The angle at B is adjacent angle of angle A

Sum of a angle and its adjacent angle is 180°.

If external angle at A is 120° then internal angle at A is 60° so The internal angle of adjacent angle of A is 120°

Q.36 The standard linear equation $ax + b = 0$

(A) $x = 0, b \neq 0$ (B) $a = 0\ b \neq 0$

(C) $a \neq 0, b \neq 0$ (D) None of these

Solution:

Definition: The standard or ideal form of a linear equation with one verifiable of power one is $ax + b = 0$ where a and b are constants x is variable and $a \neq 0$

A linear equation may be more than one variable but coefficient of verifiable not equal to zero

Q.37 Degree of the polynomial $5x^4y^2 + 3x^3 + 5y$ is:

(A) 4 (B) 3

(C) 6 (D) None of these

Solution:

The greatest sum of power in a single term of equation.

There are three terms in this equation

$5x^4y^2$ exponent are 4+2=6

$3x^3$exponent are 3

5y exponent is 1

So greatest is 6 so it is degree.

Q.38 A polynomial is divided by its factor, the remainder is equal to:

(A) 0

(B) 2

(C) A non negative number

(D) None of these

Solution:

Factor Theorem: The polynomial

$x-a$ is a factor of polynomial $f(x) \Leftrightarrow f(a)=0$

Q.39 The radius and the tangents of a circle at point of contact are

(A) perpendicular (B) Parallel

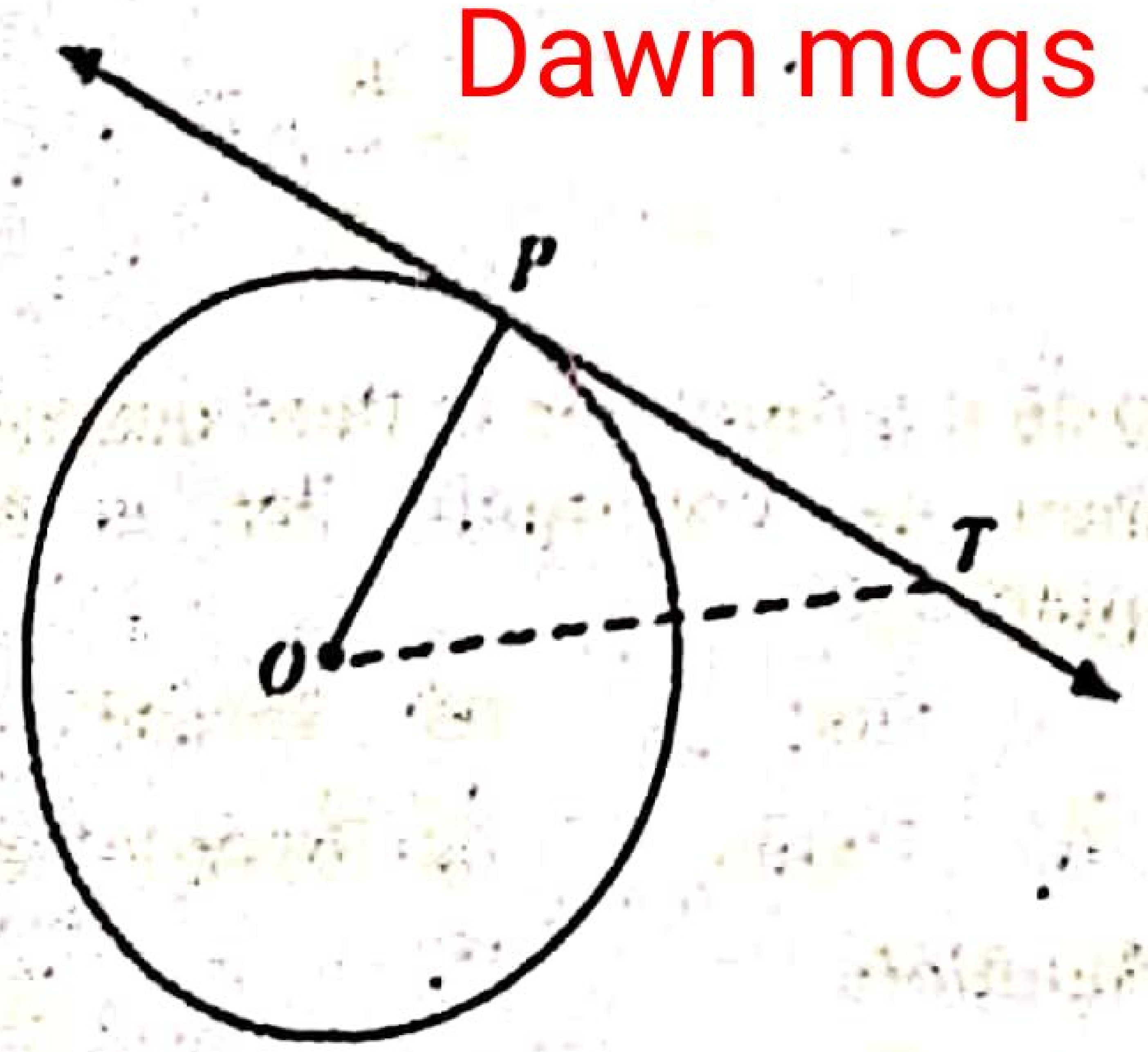
(C) Not parallel (D) None of these

Solution:

A tangent to a circle is a straight line which touches the circle at only one point. The point is called point of tangency. T is tangent OP radius of circle

$$T \perp OP$$

Dawn mcqs



Q.40 The value of external angle of a regular octagon:

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$

(C) $\frac{\pi}{7}$ (D) None of these

Solution:

Exterior or external angle polygon $= \frac{360°}{\text{no of total sides}}$

Interior angle= 180° − exterior angle

No. of side of Octagon= 8

Exterior or external angle octagon = $\frac{360°}{8} = \frac{2\pi}{8} = \frac{\pi}{4}$

Q.41 $\lim_{x\to 0} \frac{\sin 2x}{x}$ **is:**

(A) 2 (B) 1

(C) ∞ (D) −1

Solution:

$$\lim_{x\to 0} \frac{\sin 2x}{x} \times \frac{2}{2}$$

$$2\lim_{x\to 0} \frac{\sin 2x}{2x} = 2(1) = 2$$

Remember

$$\lim_{x\to\infty} \frac{\sin x}{x} = 0$$

& $\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{\tan x}{x} = 1$

Q.42 If f(x) = is, then range of f is:

(A) R (B) R − {1,2}

(C) R* (D) none of these

Solution:

Q.43

Q.44

(A) (B) logx + c

(C) x + c (D) None of these

Q.45 The area bounded by parabola $y^2 = 4ax$ and its Latus rectum is:

(A) $\frac{2}{3}a^2$ (B) $\frac{8}{3}a^2$

(C) $\frac{4}{3}a^2$ (D) None of these

Solution:

$$y = \sqrt{4ax}$$

$$\text{Area required} = 2 \times \int_0^a y\,dx$$

$$= 2\int_0^a \sqrt{4ax}\,dx$$

$$= 2\int_0^a \sqrt{4a}\sqrt{x}\,dx$$

$$= 2\sqrt{4a}\int_0^a \sqrt{x}\,dx$$

$$= 4\sqrt{a}\int_0^a \sqrt{x}\,dx$$

$$= 4\sqrt{a}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^a$$

$$= 4\sqrt{a} \times \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^a$$

$$= \frac{8}{3}\sqrt{a}\left[a^{\frac{3}{2}} - 0\right]$$

$$= \frac{8}{3}\sqrt{a}\,a^{\frac{3}{2}}$$

$$= \frac{8}{3}a^{\frac{1}{2}}\,a^{\frac{3}{2}}$$

$$= \frac{8}{3}a^{\frac{1}{2}+\frac{3}{2}}$$

$$= \frac{8}{3}a^{\frac{4}{2}}$$

$$= \frac{8}{3}a^2$$

Q.46 If a point lies in third quadrant then its conjugate lies in the quadrant

(A) First (B) Second

(C) Third (D) None of these

Solution:

In first quadrant(x, y)

In second quadrant (−x, y) In first quadrant(x, y)

In third quadrant(−x, −y)

In first quadrant(x, −y)

As point in third quadrant (−x, −y) and its conjugate is (−x, y) in second quadrant.

Q.47 The centre of the sphere $x^2 + y^2 + z^2 + 2x + 4y + 9z + 13 = 0$ is the point:

(A) $(0,0,0)$ (B) $\left(-1,-2,\frac{9}{2}\right)$

(C) $(1,2,3)$ (D) None of these

Solution:

$(x^2+2x)+(y^2+4y)+(z^2+9z)$
$=-13$

Completing square of x, y, z

$(x^2+2x+1)+(y^2+4y+4)+\left(z^2+9z+\frac{81}{4}\right)=-13+1^2+2^2+\left(\frac{9}{2}\right)^2$

$$(x+1)^2+(y+2)^2+\left(z+\frac{9}{2}\right)^2$$
$$=\frac{-52+4+16+81}{4}$$

$$(x+1)^2+(y+2)^2+\left(z+\frac{9}{2}\right)^2=\frac{49}{4}$$

Centre $\left(-1,-2,-\frac{9}{2}\right)$

Radius $\frac{7}{2}$

Q.48 Equation of

(A) Sphere (B) Hyperboloid

(C) Ellipsoid (D) None of these

Solution:

Ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

If $a=b=c$ then sphere

Hyperboloid of one sheet

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$

Hyperboloid of two sheet

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$

Elliptic Paraboloid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=z$$

Hyperbolic Paraboloid

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=z$$

Q.49 The given lines are

(A) Difference line

(B) Parallel lines

(C) Perpendicular lines

(D) None of these

Solution:

1.Angle Between Two Line:

Let $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$

and $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ be to lines then

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

Perpendicularity: lines are perpendicular if $a_1a_2+b_1b_2+c_1c_2=0$

Parallelism: The lines are parallel if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

2.Lines of type $ay+bx=c$

If $l_1: a_1y+b_1x=c_1$ and $l_2: a_2y+b_2x=c_2$ be two lines the fine the slope $\frac{dy}{dx}$ let m_1 slope of line l_1 and m_2 slope of l_2

If $m_1\cdot\frac{1}{m_2}=-1$ then line are perpendicular

If $m_1=m_2$ then line are parallel

Q.50 The curve $x^4+y^4-2x^2-2y^2+1=0$ has singular points

(A) Two (B) Three

(C) four (D) None of these

Solution:

As we know that a polynomial of degree n can have maximum n roots

Q.51 The asymptote to the curve $y = x^2$

(A) $y = 0$ (B) $x = 0$

(C) $y = x$ (D) None of these

Solution:

Given equation is $y - x^2 = 0$

For horizontal asymptote equate coefficient of higher power of x

$1 = 0$ (not possible) $\Rightarrow$ no horizontal asymptotes

For vertical asymptote equate coefficient of higher power of y

$1 = 0$ (not possible) $\Rightarrow$ no vertical asymptotes

Q.52 The equation $ax^2 + by^2 + 2gx + 2fy + c = 0$ represents:

(A) General equation of parabola

(B) General equation of ellipse

(C) Hyperbola

(D) None of these

Solution:

<u>Conic section</u>

The equation of conic section is represented by the general equation of 2nd degree

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ ---------1

And discriminent above equation is given by $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

Case -1 when $\Delta = 0$

In this case equation 1 represents degenerated conic the nature as given below

S.No.	Condition	Nature of the conic
1	$\Delta = 0$ and $ab - h^2 = 0$	A pair of coincident straight line
2	$\Delta = 0$ and $ab - h^2 < 0$	A pair of intersecting striaght line
3	$\Delta = 0$ and $ab - h^2 > 0$	A point

Case -2 when $\Delta \neq 0$

In this case equation 1 represents the nature of the conic degenerated section as given below

S.No	Condition	Nature of the conic
1	$\Delta \neq 0, h = 0, a = b$	A circle
2	$\Delta \neq 0, ab - h^2 = 0$	A parabola
3	$\Delta \neq 0, ab - h^2 > 0$	An Ellipse
4	$\Delta \neq 0, ab - h^2 < 0$	A hyperbola
5	$\Delta \neq 0, ab - h^2 < 0$ $a + b = 0$	A rectangular hyperbola

Q.53 Angle between the straight lines $N: \frac{x-1}{1} = \frac{z+1}{1}, M: \frac{x-2}{2} = \frac{y-2}{-3} = \frac{z+1}{3}$

(A) 40° (B) 40.1°

(C) 40.2° (D) None of these

Solution:

1.Angle Between Two Line:

Let $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and

$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ be to lines then

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\theta = \cos^{-1}\left(\frac{5}{\sqrt{44}}\right) = 40.1°$$

Q.54 A cyclic group of order n is generated by:

(A) n elements (B) $(n-1)$ elements

(C) One element (D) None of these

Solution:

Theorem: Every group of prime order is cyclic.

And a cyclic group generated by a single element.

Q.55 let G be a group of order 13, then

(A) G is not cyclic

(B) G is non abelian

(C) G is commutative

(D) None of these

Solution:

Theorem: Every group of prime order is cyclic.

Theorem: Every cyclic group is abelian

Abelian group is commutative

Q.56 Identify elements in a cyclic group is:

(A) Infinite (B) Unique

(C) finite (D) None of these

Solution:

Theorem: There is a unique element in a group.

Q.57 $\forall\, a, b \in G, (ab)^2 = a^2b^2$ **then:**

(A) G is cyclic

(B) G may be abelian

(C) G is abelian

(D) None of these

Solution:

$$abab = aabb$$

By Cancellation law

$$ba = ab \Rightarrow \text{abelian}$$

Q.58 Which of the following statement is correct?

(A) A group can have more than one identity element

(B) The null set can be considered be a group

(C) The set of all real number is a group with respect to subtraction

(D) To each elements of a group there corresponds only one inverse element

Solution:

To each elements of a group, there corresponds only one inverse element

Q.59 The unit matrix of order n has rank

(A) Zero (B) N

(C) 1 (D) None of these

Solution:

Unit matrix of n order consist of n column with diagonal element 1 so rank is n

Q.60 If determinant $|A| = 2$, then ; $|A^5| =$

(A) $|A^4| = 12$ (B) $|A^5| = 32$

(C) $|A^6| = 60$ (D) None of these

$$|A|^n = |A^n|$$

$$|A^5| = |A|^5 = 2^5 = 32$$

Q.61 If $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ is invertible under matrix multiplication, then its inverse is:

(A) $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ (B) $\frac{1}{a^2+b^2}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

(C) $\frac{1}{a^2+b^2}\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ (D) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$

Solution:

A said to be invertible matrix if A^{-1} exist

$$A^{-1} = \frac{adjA}{|A|}$$

$$\begin{bmatrix} a & -b \\ b & a \end{bmatrix}^{-1} = \frac{1}{a^2+b^2}\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$$

Q.62 A homogeneous system of linear equation has a non-trivial solution if:

(A) The number of unknowns exceeds the number of equations

(B) The number of equations exceeds the number of unknowns

(C) The number of unknowns proceeds the number of equations

(D) None of these

Solution:

Let a system of m homogeneous linear equation in n variable

And $AX = O$ where O is zero matrix of order $m \times 1$

A system of homogeneous equations is always consistent.

If $m < n$ then system has non-trivial solution.

Q.63 A square matrix A such that $A^2 = A$ is called:

(A) Involutory (B) Idempotent

(C) Nilpotent (D) Symmetric

Solution:

Involutory: $A^2 = I$

Idempotent: $A^2 = A$

Nilpotent: $A^n = 0$

Q.64 Co-factors of the elements of the second of row of the determinant $\begin{bmatrix} 1 & 2 & 3 \\ -4 & 3 & 6 \\ 2 & -7 & 9 \end{bmatrix}$ are:

(A) $-39, 3, 11$ (B) $6, 5, 4$

(C) $3, 11, -39$ (D) $13, 1, 3$

Solution:

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 3 & 6 \\ 2 & -7 & 9 \end{bmatrix}$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 2 & 3 \\ -7 & 9 \end{vmatrix} = -1(39)$$
$$= -39$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix}1 & 3\\ 2 & 9\end{vmatrix} = (3) = 3$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix}1 & 2\\ 2 & -7\end{vmatrix} = -1(-11) = 11$$

Q.65 A system of m homogeneous linear equation $AX = 0$ in n variables has non-trivial solutions if and only if

(A) Rank $A = n$ (B) Rank $A < n$

(C) Rank $A > n$ (D) None of these.

Solution:

Theorem: A system of m homogeneous linear equation $AX = 0$ in n variables has non-trivial solutions if and only if Rank $A < n$

Q.66 The intersection of two infinite sets is:

(A) Always infinite

(B) Always finite

(C) May not be infinite

(D) None of these

Solution:

May not be infinite

Q.67 Least upper bound of set, if exists is:

(A) Infinite (B) Finite

(C) Unique (D) Always friction

Solution:

Definition: The smallest of all upper bound of a set of numbers is called least upper bound is unique

Q.68 The greatest lower bound of a set

(A) Always belong to set

(B) Not belong to the set

(C) May or may not belong to the set

(D) None of these

Solution:

$\left\{\frac{1}{n}, n \in N\right\}$ g.l.b 0 not belong to set

$[1, 5]$ the g.l.b 1 belong to set

Q.6 If $a, b \in R$, then:

(A). $|ab| > |a| \cdot |b|$

(B) $|ab| \geq |a| \cdot |b|$

(C) $|ab| < |a| \cdot |b|$

(D) $|ab| \leq |a| \cdot |b|$

Solution:

$$|ab| \leq |a| \cdot |b|$$

Q.70 A convergent sequence converges to:

(A) A unique limit (B) Many limits

(C) Any two limits (D) None of these

Solution:

Theorem: A convergent sequence has a unique limit

Q.71 Every pair of real numbers a and b satisfies one, and only one, of the condition:

$a > b, a = b, b > a$. This property of real real numbers is known as the:

(A) Transitive law

(B) Associative law

(C) Trichotomy law

(D) Commutative law

Solution:

Transitive law: $a = b, b = c, \Rightarrow a = c$

Associative law: $a(bc) = (bc)a$

Trichotomy law: $a > b, a = b, b > a$

Commutative law: $ab = ba$

Q.72 $1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \frac{z^4}{4!} + \cdots$

converges to:

(A) e^z (B) e^{-z}

(C) $-z^{e^2}$ (D) None of these

Solution:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$e^z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \frac{z^4}{4!} + \cdots$$

Q.73 The degree of the differential equation of all tangent lines to parabola $y^2 = 4ax$ is:

(A) 3 (B) 1

(C) 2 (D) 4

Solution:

$y^2 = 4xa$---------1

Differentiate w.r.t x

$$2y \cdot y' = 4a$$

$$y \cdot y' = 2a$$

Again, differentiate w.r.t x,

$$y \cdot y'' + y' \cdot y' = 0$$

$$y \cdot y'' + y'^2 = 0$$

Definition: The highest number of the derivative in differential equation is called **order of differential equation.**

So the highest number of derivative is 2 so degree is 2

Q.74 The order of differential equation defined as:

(A) The highest degree of variable

(B) The highest order highest derivative

(C) The power of variable in the solution

(D) None of these

Solution:

Definition: The highest number of the derivative in differential equation is called **order of differential equation.**

Definition: The power of the highest derivative in differential equation is called **degree of differential equation.**

Q.75 The selling price of an article is 118 and profit is 50%. What would be the cost price

(A) 78 (B) 77.66

(C) 80 (D) 78.6

Solution:

Let cost price = x

Selling Price = 118

Profit % = 50%

$$\text{Profit \%} = \frac{\text{profit}}{\text{cost price}}$$

$$\frac{50}{100} = \frac{118 - x}{x}$$

$$50x + 100x = 11800$$

$$150x = 11800$$

$$x = 78.66$$

Q.76 A boat can travel the speed of $13kmh^{-1}$ in water, if the speed of the stream is $4kmh^{-1}$ at what time, the boat go 68km downstream.

(A) 3hours (B) 4 hours

(C) 5 hours (D) 2 hours

Solution:

$$S = vt$$

For upstream $v = v - v_{steam}$

For downstream $v = v + v_{steam}$

$$v = 13 + 4 = 14kmh^{-1}$$

$$t = \frac{S}{v}$$

$$t = \frac{68}{17} = 4h$$

Q.77 The differential equation $Mdx + Ndy$ is define as an exact differential equation if:

(A) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$ (B) $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

(C) $\frac{\partial M}{\partial y} = -\frac{\partial N}{\partial x}$ (D) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$Mdx + Ndy$ is called exact differential equation if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Q.78 The differential equation $[1 + (y')^2]^{\frac{1}{2}} = y''$ has order and degree respectively:

(A) 2,1 (B) 2, 2

(C) 1, 2 (D) None of these

Solution:

Definition: The highest number of the derivative in differential equation is called **order of differential equation.**

Definition: The power of the highest derivative in differential equation is called **degree of differential equation.**

$$[1 + (y')]^{\frac{1}{2}} = y''$$

$$1 + y' = (y'')^2$$

Order 2 and degree 2

Q.79 The differential equation $\frac{dy}{dx} = \frac{x^2+xy+4y}{4x^2+y^2}$ is:

(A) Exact (B) Homogeneous

(C) Cauchy (D) None of these

Solution:

$Mdx + Ndy$ is called exact differential equation if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

The differential equation $(a_1x + b_1y + c_1)dx + (a_2x + b_2y + c_2)$ is called homogeneous if $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

Given differential equation not exact not homogeneous

Q.80 A general solution of 3rd order differential equation contain:

(A) One constant

(B) Two constants

(C) Three constants

(D) None of these

Solution:

For 3rd order differential equation, we take 3 time integral so 3 constants added